This test is open notes. Answer all questions and PLEASE BE NEAT. If possible, write out the formulae before plugging in the numbers. If the questions require assumptions, state and justify them clearly! The back of this sheet has some physical constants that may (or may not) be useful.

1. (10 pts) You listen to a colloquium by Jason Wright about Tabby's star, an F-type main sequence object with unusual light fluctuations that have been explained in the news media as possibly being due to orbiting alien megastructures. You get excited about this topic, and want the Hobby-Eberly Telescope (HET) to observe it, in order to correlate the photometric variations with spectroscopic changes. Tabby's star has a right ascension of $\alpha(2000) = 20^{\mathrm{h}}\ 06^{\mathrm{m}}\ 15.4^{\mathrm{s}}$ and a declination of $\delta(2000) = +44^\circ\ 27'\ 25''$. The next proposal deadline for the HET is November 6, and it is for observations during the December through March trimester. Should you hurry up and write a proposal for observations during this time?

Tabby's star is in the northern part of the sky, so there's no problem reaching it with the HET. However, its right ascension is at $\sim 20^{\mathrm{h}}$. On the autumnal equinox, $0^{\mathrm{h}}$ is on the meridian at midnight, and the sky progresses about 2 hours per month. This means that Tabby's star crosses the meridian at midnight around July 21. Roughly 4.5 months later, in the beginning of December, the star will cross the meridian 9 hours earlier at 3 in the afternoon: by sunset the object will be setting. At the end of March, the star will cross the meridian near 8 a.m., again making it unobservable for almost the entire night. One therefore shouldn't propose to observe it during the December through March trimester.

2. Gaia measures the parallax angle of a $m = 15.5$ F7 star to be $p = 0.1$ milliarcsec.

   a) (5 pts) What type of star is the object likely to be? Is fusion occurring in the star's core? If so, what element is being fused (and to what)?

The parallax puts the object at a distance of $1/p = 10000$ pc. The absolute magnitude of the star is then

$$M = m - 5\log(d/10) = +0.5$$

A F-star with an absolute magnitude of $+0.5$ lies substantially above the main sequence, in the "Giant" region of the diagram. More specifically, it is in the region occupied by horizontal branch stars. The star is therefore burning helium to carbon and oxygen in its core.

   b) (5 pts) Follow-up observations demonstrate that the object is a single-line spectroscopic binary in a roughly circular orbit. Although there's not much information on the secondary, analysis suggests that the separation between the two stars is $\sim 0.5$ A.U. Has mass transfer played a role in the evolution of the system? How do you know? (Think it through.)

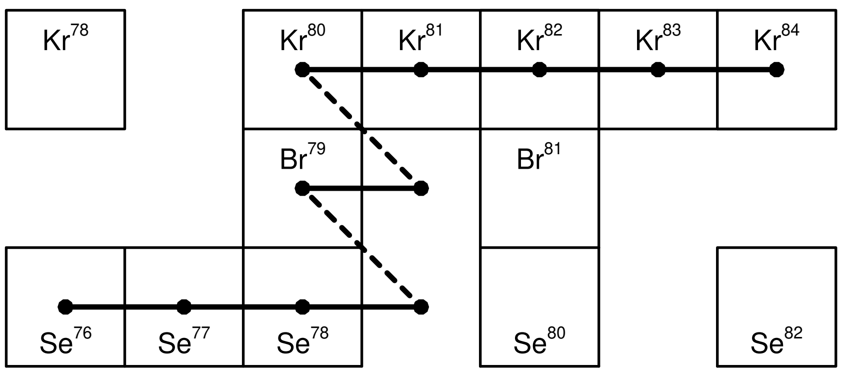
Since the primary of the system is a horizontal branch star, it likely began its life on the lower part of the main sequence, with a mass $M < 2M_{\odot}$. If this is correct, then, while at the top of the RGB, its size was $\sim 100R_{\odot}$, or $\sim 0.5$ A.U. Mass transfer must have occurred. On the other hand, if the star began life with $\mathcal{M} > 2\mathcal{M}_{\odot}$, then in order to currently be a a horizontal branch star, it must have lost an abnormal amount of mass, i.e., via mass transfer. So this system must have evolved via binary evolution.

3. Below is a list of the stable isotopes of selenium, bromine, and krypton.

| Element | # protons | Stable Isotopes |
|---|---|---|
| Selenium | 34 | 76, 77, 78, 80, 82 |
| Bromine | 35 | 79, 81 |
| Krypton | 36 | 78, 80, 81, 82, 83, 84 |

a) (5 pts) Of the isotopes listed above, which would you expect to be cosmically least abundant? Why?

The diagram below shows the location of the stable isotopes in the atomic number versus neutron diagram.




$^{78}$Kr does not lie on the $s$-process chain, and it is shielded from the $r$-process. It can therefore only be formed via the $p$-process. Since this requires overcoming a substantial electrostatic repulsion, we can expect that it is quite rare. (And indeed it is.)

b) (5 pts) How is $^{81}$Br produced? How is $^{79}$Br produced?

According to the table, $^{80}$Br is unstable (it has a half-life of 17.6 min), so $^{81}$Br cannot be created via the $s$-process. Conversely, there is nothing stopping it from being the product of a series of $\beta$-decays from a very neutron-rich isotope. It is therefore an $r$-process element. $^{79}$Br can be made via the $s$-process and the $r$-process.

4. (10 pts) A 2 arcsec diameter optical fiber feeds a $R = 2500$ spectrograph attached to a 2-meter telescope. You use the instrument to observe a source that has an emission line at 5000 Å. The monochromatic flux in the emission line is $F = 10^{-16}$ ergs cm$^{-2}$ s$^{-1}$, while the object's underlying continuum is negligible. Meanwhile, the sky background is $m_{AB} = 20$ mag arcsec$^{-2}$. If the system throughput, $\epsilon = 10\%$, what signal-to-noise do you obtain with a 1/2 hour exposure?

The total number of counts from the object is

$$N = F\left(\frac{\lambda}{hc}\right)\left(\frac{\pi D^2}{4}\right)\epsilon\,\Delta t = 142 \text{ counts}$$

where $D$ is the telescope diameter and $\Delta t$ is the exposure time. For the sky background, we also need the area of sky being observed, and the number of Angstroms the emission line is spread over. The former is the area of the fiber. Since the diameter of the fiber is $r = 2$ arcsec,

$$\Delta A = \pi\frac{r^2}{4}$$

The latter is given by the resolution of the spectrograph

$$R = \frac{\lambda}{\Delta\lambda} \implies \Delta\lambda = \frac{\lambda}{R} = 2 \text{ Å}$$

Thus the sky background of the object is

$$n = 3.63 \times 10^{-20}\,10^{-m_{AB}/2.5}\left(\frac{c}{\lambda^2}\right)\left(\frac{\lambda}{hc}\right)\left(\frac{\pi D^2}{4}\right)\left(\frac{\pi r^2}{4}\right)\Delta\lambda\,\epsilon\,\Delta t = 389 \text{ counts}$$

The signal-to-noise is then

$$SNR = \frac{N}{\sqrt{N+n}} = 6.16$$

5. (10 pts) Will the Sun ever in its lifetime become as bright as a $15\,\mathcal{M}_\odot$ main sequence star? Be sure to state your assumptions and/or explain where your numbers come from.

Roughly speaking, the main sequence luminosity of a $15\,\mathcal{M}_\odot$ star is

$$\mathcal{L} \sim \left(\frac{\mathcal{M}}{\mathcal{M}_\odot}\right)^{3.5} \mathcal{L}_\odot \sim 13,000\mathcal{L}$$

The brightest the Sun will ever get will be when it is at the tip of the asymptotic giant branch. During this time, the luminosity will be (very approximately)

$$\mathcal{L}_{\mathrm{AGB}} \sim 60,000\,(\mathcal{M}_{\mathrm{core}} - 0.52)$$

with $\mathcal{M}_{\mathrm{core}}$ as the final core mass of the Sun in solar masses. Since the Sun will someday evolve into a white dwarf with a mass between 0.5 and $0.6\mathcal{M}_\odot$, its maximum luminosity will be $\sim 4800\mathcal{L}_\odot$. That is almost three times fainter than a $15\,\mathcal{M}_\odot$ main sequence star.

6. A nova erupts in the constellation of Sagittarius. This eruption lasts for about 90 days, during which time the object has an apparent bolometric magnitude of $m = 6$. Spectroscopy of the material lost from the star shows it to have an ejection velocity of $\sim$ 500 km s$^{-1}$. Two years later, the ejecta becomes (barely) resolvable at 5000 Å with the diffraction limited Hubble Space Telescope (aperture 2.4 m).

a) (5 pts) What is the approximate distance of the nova?

In time $t$, the ejecta has covered a distance of $v\,t$. If $\theta$ is the the angle subtended by this ejecta, then the distance to the object is

$$R = \frac{v\,t}{\theta}$$

The diffraction limit of the Hubble Space Telescope is

$$\theta_{\text{diff}} = 1.22\frac{\lambda}{D} = 2.54 \times 10^{-7} \text{ radians} = 0.05 \text{ arcsec}$$

Setting $\theta = \theta_{\text{diff}}$ and $t = 2$ yr yields a distance of $R \sim 4$ kpc.

b) (10 pts) Now that you know the distance to the nova, how much energy was released in the explosion? If the accretion rate prior to the explosion was $10^{-10} M_\odot$ yr$^{-1}$ and there was no "hibernation" period between explosions when accretion ceased, how long was the time between outbursts?

The absolute bolometric magnitude is related to the apparent magnitude and the distance (in parcsecs) by

$$M = m - (5 \log R/10) = -7$$

This corresponds to a luminosity of

$$M = -2.5 \log \mathcal{L}/\mathcal{L}_\odot + 4.74 \Longrightarrow \log \mathcal{L} = 4.7 \Longrightarrow \mathcal{L} \sim 2 \times 10^{38} \text{ ergs s}^{-1}$$

Consequently, over $\Delta t = 90$ days, the total energy produced is

$$E = \mathcal{L}\,\Delta t = 1.5 \times 10^{45} \text{ ergs}$$

Hydrogen fusion releases $Q_H \sim 6 \times 10^{18}$ ergs gm$^{-1}$, so during its outburst, the nova burned $\mathcal{M}_H = E/Q_H = 2.5 \times 10^{26}$ gm of hydrogen. Since hydorgen only comprises 75% of the matter accreted onto the star, the total amount of mass accreted is $\mathcal{M}_T = \mathcal{M}_H/X = 3.3 \times 10^{26}$ gm $= 1.6 \times 10^{-7}\,\mathcal{M}_\odot$. If matter accreted at a rate of $10^{-10}\,\mathcal{M}_\odot$ yr$^{-1}$, and all the hydrogen was fused in the nova explosion, then the time between outbursts was $\sim$ 1650 yr.

7. A $1M_\odot$ protostar sequence star collapses from a radius of $\sim 100R_\odot$ to $5R_\odot$.

a) (5 pts) If the star has an initial luminosity of $\sim 50L_\odot$, very roughly, how long will this collapse take?

The star will collapse on a thermal timescale, so

$$\Delta t \sim \frac{G\mathcal{M}^2}{R\mathcal{L}} \sim 2 \times 10^{13}\ \text{s} \sim 6 \times 10^5\ \text{yr}$$

b) (5 pts) What is the star's mean luminosity over this time period? (For simplicity, you may assume that the star acts as a uniform density, ideal gas.)

During the collapse, the gravitational potential of the star changes by

$$\Delta E_{\text{grav}} = \frac{3}{5} G\mathcal{M}^2 \left( \frac{1}{R_1} - \frac{1}{R_2} \right) = 4 \times 10^{47}\ \text{ergs}$$

For an ideal gas star to stay in virial equilibrium, half of this energy must go into heating the star; the other half is radiated away. So during this period, the average luminosity of the star must be

$$\mathcal{L}_T = \frac{1}{2} \frac{\Delta E_{\text{grav}}}{\Delta t} \sim 6\mathcal{L}_\odot$$